METROLOGICAL APPLICATIONS OF A PROPERTY OF STATIONARITY IN RECTANGULAR CAVITIES CONTAINING A DIELECTRIC SLAB

Christian E. EUGÈNE

Laboratory for electrical measurements and microwaves laboratory
Catholic University of Louvain at Louvain-la-Neuve, Belgium.

Abstract

A property of stationarity for the resonant frequency of a rectangular cavity in function of the thickness of a dielectric slab inserted in it, leads to a new and accurate measuring method of the permittivity of the sample.

Introduction

A rectangular microwave cavity is considered (see figure 1), which contains in its middle a low loss, homogeneous and preferably isotropic dielectric slab (filling the x-y section) and which resonates in a mode TE_{1on} (thus the electric field of which is orientated along the y-axis)

For the modes with index $n \geq 2$, it exists, on each side of the middle plane, at least one x-y plane on which the field E is null. If this plane coincides with the dielectric interface, the resonant frequency of the cavity does not change at the first order in function of the sample thickness. This property of stationarity - which according to us has not yet been considered in the litterature - presents interesting metrological consequences which will be used for the accurate (though simple) determination of the permittivity of the sample.

Characteristic relations at a stationary point

As the electric field is constant along the y-axis and orientated in this direction, the concerned structure is bidimensional and inhomogeneous on one dimension. For such a problem the eigenvalues equation and the field expressions are well known (see for instance[1]) Given on a normalized form, one has :

- eigenvalues equation

$$\frac{\tan X}{X} = \pm \frac{T}{1-T} \frac{(\cot Y)^{\pm 1}}{Y} \qquad (1)$$

(sign plus or minus according as the mode is odd or even for the n-index) where we put

$$X = \frac{\pi}{2}(1-T)\sqrt{F^2-A^2}$$

$$Y = \frac{\pi}{2} T \sqrt{\varepsilon F^2 - A^2}$$

$$F = 2d\sqrt{\varepsilon_o \mu_o}\, f; \quad A = 1\frac{d}{a}; \quad T = \frac{t}{d}$$

(ε is noted for the relative permittivity - scalar quantity as the losses are neglected)

From the measured frequency and the geometrical dimensions, this equation is easily solved for ε by means of a calculator if care is taken to extract the right root

- electric field expression (limited by rea of symmetry to the first half of the cavity)

$$E = Ey(x,z) = f(z) \sin l\pi x/a$$

on air portion : $f(z) = \sin \frac{2X}{(1-T)d} z \qquad (2)$

on dielectric portion

$$f(z) = \frac{T}{1-T} \frac{X}{Y} \cos X \sin \frac{2Y}{Td}\left(z - \frac{1-T}{2}d\right) + \sin X \cos \frac{2Y}{Td}\left(z - \frac{1-T}{2}d\right) \qquad (2)$$

The stationary point (noted with the subscript s for each related quantity) is characterized by f(z) = 0 at the dielectric interface. This hields

$X_s = k\pi$ (k, integer, is the order of zero of electric field numbered from z = 0) which, put in the eigenvalues equation (1) gives a second stationary relation :

$$Y_s = (n-2k)\frac{\pi}{2}$$

These two relations allow, at a stationary point, to relate, two by two, the resonant frequency, the sample thickness, and its permittivity.

$$F_s = \left(A^2 + \frac{4k^2}{(1-T_s)^2}\right)^{1/2} \qquad (3a)$$

$$\varepsilon_s = \frac{A^2 + (n-2k)^2/T_s^2}{A^2 + 4k^2/(1-T_s)^2} \qquad (3b)$$

$$\varepsilon_s = \frac{1}{F_s^2}\left(A^2 + \left(\frac{n-2k}{1-\frac{2k}{\sqrt{F_s^2-A^2}}}\right)^2\right) \qquad (3c)$$

These stationary formulas can be considered as very simple.
As an example, on figure 2 are plotted, for different values of ε, the F-T curves of a cavity with d=a and resonating in the mode TE_{103}. On the same diagram is superposed the stationary F_s-T_s curve (formula 3a) which crosses the formers at an inflexion point with horizontal tangent as it can be shown.

In the next paragraphs, are described two methods of determining the permittivity of the sample and which proceed from the property of stationarity.

First method : determination of the permittivity by frequency measurement around a stationary point

The advantage to solve the eigenvalues - equation in the vicinity of a stationary point is clearly the larger tolerance on the sample thickness which is permitted for a given error on ε. Such an information can be presented graphically as on figure 3 which appeals several comments.

(1) The upper and lower limit curves give strictly the tolerances over T to reach an error of ± 1% over ε at a stationary point. For instance for a permittivity ε_c, its determination will be accurate to 1% if we take a thickness T_1 (stationary value) with tolerances (T_2-T_1) and (T_1-T_3). Besides this particular interpretation, this diagram has a more general scope.

(2) The tolerances over T are still shown when one diverges from the stationary point. For the same thickness T_1 but with a permittivity ε_b the two tolerances become respectively (T_4-T_1) and (T_1-T_5) (This comment is no more valid when b comes near a). If the permittivity is estimated beforehand between certain limits, one can thus fix the most appropriate thickness and the corresponding acceptable tolerances are immediatly given.

(3) The same diagram gives finally the range of validity of the stationary formula $F_s-\varepsilon_s$(3a) when used outside a stationary point. For the thickness T_1 and without going beyond an error of ± 1%, we can apply this formula for an ε-range between ε_a and ε_d (This range is reduced in function of the tolerance over T_1). As we can see this range is very high (typically between 70% and 100% of the corresponding stationary value of ε). Using formula $F_s-\varepsilon_s$ gives thus a mean very quick and accurate in large limits to determine the permittivity.

In spite of the advantages mentioned above of staying in the vicinity of a stationary point, a study of sensitivity shows that, in connection with the accuracy on ε, three items are critical.

(1) The accuracy on the measured frequency

For pratical cases, the admissible uncertainty over f giving an error on ε lower than one percent is a few parts in ten thousands. The use of a cavity wavemeter can be very limiting in this sense. It is to be noted that the measured frequency must be corrected for taking into account the coupling holes and cavity-Q effects.

(2) The accuracy on the geometrical dimensions of the cavity.

A few hundreths of mm are normal in X-band, always to reach one percent accuracy on ε.

(3) The interstice between the sample and the walls perpendicular to the electric field

The same order of magnitude than in previous item is here of application. It should be noted that this problem was solved by the classical method of perturbation (see for instance [2]) consisting to assimilate the fields perturbated by the interstice to unperturbated fields. For reasonable gaps, the theoretical results are closely confirmed by experimentation.

The previous statements lead us to propose an other method of determination of ε, using the stationary property in a different manner.

Second method : determination of permittivity by solving the ε_s-T_s formula at a stationary point.

In the previous method one solved the eigenvalues equation (or quicker the ε_s-F_s formula) around a stationary point. It is now proposed to solve preferably the ε_s-T_s formula (3b), this time exactly on a stationary point. Several considerations plead for this method.

(1) The frequency disappears from the problem

(2) As it is shown by a study of sensitivity, the accuracy on the geometrical dimensions is here much less critical than with the first method. A gain of one order of magnitude on the tolerances is typical (this time, the tolerance over the sample thickness is similar to that over the cavity dimensions).

(3) The stationary thickness T_s can be known with great accuracy. It is evidently excluded to adjust the sample thickness to its stationary value by the observation of a stationarity in the frequency (the sensitivity of this method should be desastrous). The proposed procedure derives from the remarkable property that, at a stationary point, the frequency does not change for any shift of the slab from the center.

If now one swerves from the stationary thickness, one proves the following result :

$$df/f = a\theta\delta^2 \qquad (4)$$

where we have

- df/f = relative frequency shift due to uncentering of the sample
- Θ = relative deviation of T from its stationary valve
- δ = uncentering of the sample (δ is supposed small)
- a = constant of proportionality depending on the mode and the geometrical dimensions.

For a given uncentering, the frequency shift is thus proportional to the thickness deviation relative to its stationary value. The procedure required for the determination of T_s proceeds immediatly from this statement.

(4) Finally, due to the differential character of this method of determination of T_s, one shows that the interstice quoted above has an effect on the result only at the second order.

The previous comments show the superiority of this second method over the first one regarding the accuracy. The price to be paid is a procedure slightly longer.

Some results of measurements

Comparative measurements were made in X-band on two standard samples, respectively of pure fused silica ($\varepsilon \sim 3,8$) and crosslinked polymer ($\varepsilon \sim 2,5$). A discrepancy smaller than 1% between the results obtained by the two methods was observed for both samples. The second method is however more reliable as already said and gives an uncertainty estimated to a few pro-mille. Otherwise the agreement between our results and the attested value of our samples (cut from stocks issued from an eminent U.S. research laboratory) stays also within one pro-cent. Machining and environmental conditions could explain partially this difference. Further work is here to be done.

Conclusions

Starting from a stationary property encountered in rectangular cavities containing a dielectric slab, two methods were developed for the determination of the permittivity of a dielectric sample. The first one is characterized by simplicity and rapidity, the second by accuracy (although still simple). Such a rectangular sample shape is interesting because it corresponds to material structures used in many practical devices. Furthermore, the rectangular cavity is very easy to build from a commercial wave guide.

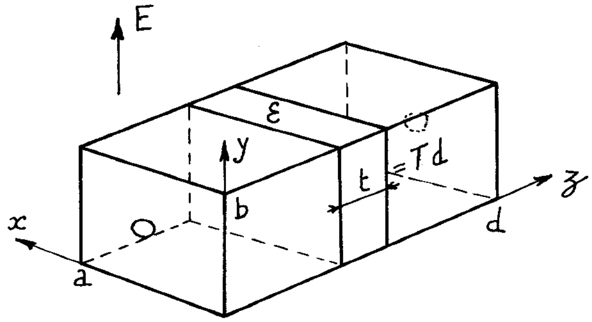
References

(1) R.Collin : Field Theory of Guided Waves Mc Graw Hill 1960 Chap. 6

(2) Harrington : Time-harmonic Electromagnetic Fields Mc Graw Hill 1961 Chap. 7

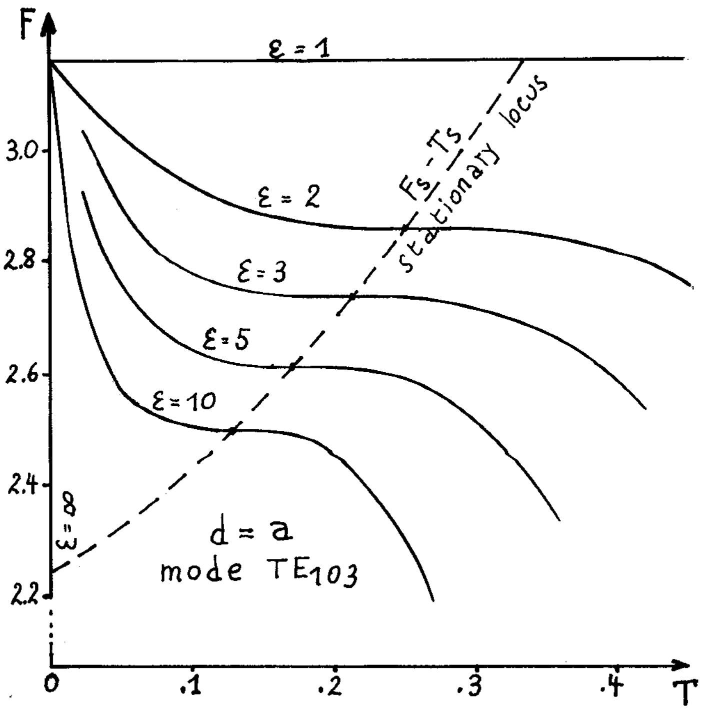
Acknowledgments

The author wishes to thank Prof. A. VANDER VORST and Prof. P. DELOGNE of University of Louvain for useful discussion. He is also indebted to Dr. W.B. WESTPHAL of M.I.T. which kindly supplied him with the standard samples.

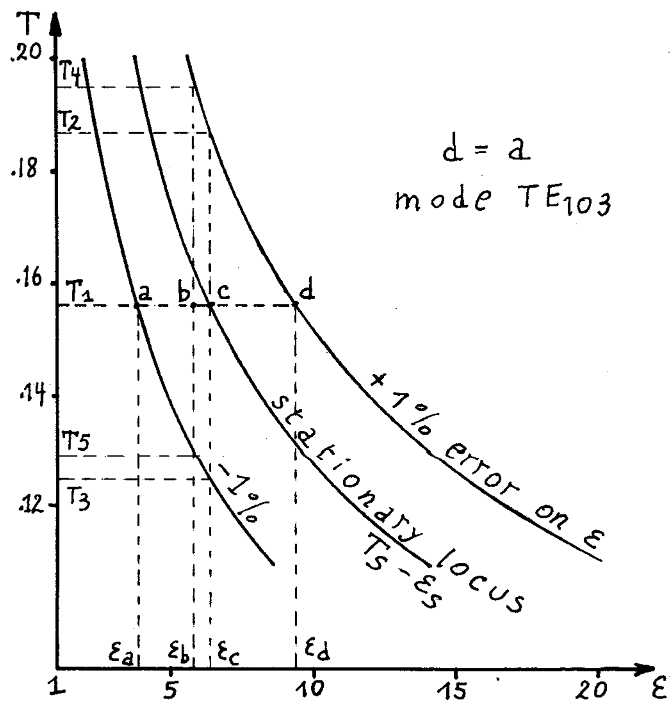


- F I G 1 -

Rectangular cavity with the slab in its center (the coupling holes are shown)



- F I G 2 -



- F I G 3 -